SOME ASPECTS OF PORTFOLIO OPTIMIZATION

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Modern portfolio theory is a theory of investment which attempts to maximize portfolio expected return for a given amount of portfolio risk, or equivalently minimize risk for a given level of expected return, by carefully choosing the proportions of various assets. This theory is widely used in practice in the financial industry and several of its creators (Markowitz, Sharpe) won a Nobel memorial prize for the theory.

Modern portfolio theory is a mathematical formulation of the concept of diversification in investing, with the aim of selecting a collection of investment assets that has collectively lower risk than any individual asset. That this is possible can be seen intuitively because different types of assets often change in value in opposite ways. For example, to the extent prices in the stock market move differently from prices in the bond market, a collection of both types of assets can in theory face lower overall risk than either individually. But diversification lowers risk even if assets returns are not negatively correlated—indeed, even if they are positively correlated.

More technically, modern portfolio theory models an assets return as a normally distributed function (or more generally as an elliptically distributed random variable), defines risk as the standard deviation of return, and models a portfolio as a weighted combination of assets, so that the return of a portfolio is the weighted combination of the assets returns. By combining different assets whose returns are not perfectly positively correlated, investors seek to reduce the total variance of the portfolio return.

We can see the advantages of portfolio optimization on the Russian Trade System data example.

We analyzed stock prices data of the 6 companies for one year time period. Rates of return of these companies are given in the table below.

Table 1 – Financial instruments' profitability

Financial instrument	Rate of return, %
URKA	47,1755
ROSN	17,5588
GAZP	21,3941
GMKN	28,6155
LKOH	10,9937
SBER	24,4289

There are two main different approaches for portfolio optimization: Markowitz and Sharpe models.

If we set the required rate of return at 25 %, we can describe stock distribution of the portfolio and its risk in the next table.

Table 2 – Risk minimization of the portfolio

Shares of securities in portfolio	Equally distributed portfolio	Optimal portfolio by Markowitz	Optimal portfolio by Sharpe
1 (URKA)	0,16666	0,11708	0,06991
2 (ROSN)	0,16666	0,00000	0,00000
3 (GAZP)	0,16666	0,00000	0,00000
4 (GMKN)	0,16666	0,00000	0,05317
5 (LKOH)	0,16666	0,15349	0,09020
6 (SBER)	0,16666	0,72942	0,78672
Portfolio variance, %	12,07	9,65	10,04

The practical aprobation of these two models shows that investor will receive better results using Markowitz model.

Dependence between risk and rate of return of the portfolio is graphically represented in the next picture.

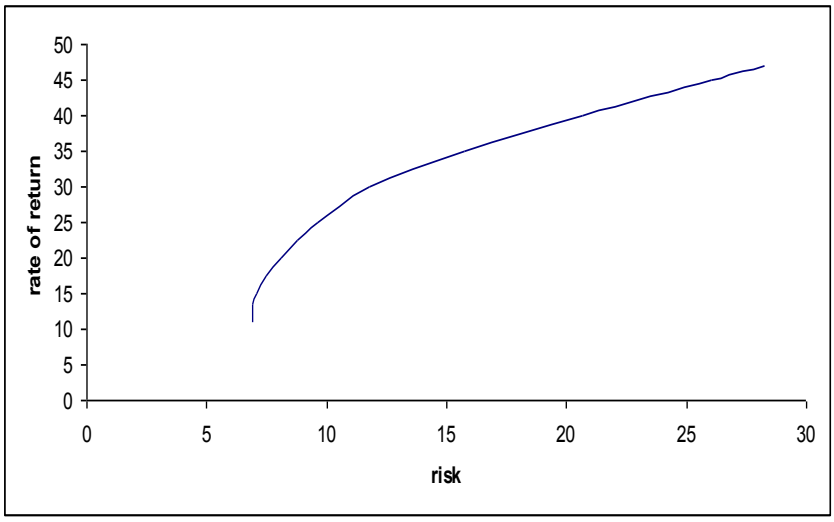

Figure1 – Dependence between the portfolio risk and profitability